

8. EXAMPLE OF A JORDAN NORMAL FORM
Part II

$$A = \begin{pmatrix} 4 & 2 & -1 \\ 0 & 2 & 0 \\ 4 & 4 & 0 \end{pmatrix} \quad P_A(x) = (x-2)^3 \quad S^{-1}AS = J \quad S = (v_1 \ v_2 \ v_3)$$

$$(A - 2 \cdot E)v = 0 \Leftrightarrow \begin{pmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 4 & 4 & -2 \end{pmatrix} \cdot v = 0 \Leftrightarrow v = \begin{pmatrix} a \\ b \\ 2a+2b \end{pmatrix}$$

$$\Rightarrow \text{Eig}(A, 2) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$$(A - 2 \cdot E) \cdot v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 4 & 4 & -2 \end{pmatrix} \cdot v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad \begin{matrix} \swarrow \\ \searrow \end{matrix}$$

$$\Rightarrow (A - 2 \cdot E) \cdot v_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 4 & 4 & -2 \end{pmatrix} v_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow v_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 2 & -1 & 2 \end{pmatrix} \quad S^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \Rightarrow S^{-1} \cdot A \cdot S = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$