



## One Health: Connecting Humans, Animals and the Environment Video Transcript

### Solving the formula, part 1: Introducing the Matrix

[Jakob Zinsstag]: In this video, we will do some mathematics. But don't worry, if you don't understand the logic behind this at once, important is only to know that we use the following formulas and matrices to visualise growth rates of cattle and to understand how diseases can influence the population of a herd. And please feel free to pause the video at every moment to have a look at each individual slide. So let's start with some definitions. Animal populations can be represented using the two aspects: age and sex.

Here you see a representation of a cattle population that is divided in females and males and three age classes: newborn young ones, subadult juveniles which are sexually immature, and adult ones, cows and bulls who reproduce. The arrows indicate the birth and death of animals and the transitions between age classes. The calculation of the dynamics of such a population is straightforward.

The different classes of a population can be considered as a vector called the population vector  $N$ . Here we look only at the female group. The different age classes can be represented as  $Y$  for young ones,  $S$  for subadults, and  $A$  for adults. Further, we have a so-called projection matrix that describes the demographic behaviour of the population. Such a projection matrix has a typical structure. It consists of the birth rate, the survival rate of the young animals, and the persistence of the subadults and adults. In the top right corner, you have the birth rate  $b$ . On the diagonal, you have the rate of persistence in the same class: for subadults  $p_s$ , and for adults  $p_a$ . This is the inverse of the duration in that class. On the first subdiagonal is the survival rate of the young ones,  $s_y$ , and the subadult,  $s_s$ .

If we want to project a population  $N$  at a certain time  $t$  plus 1, we can take the population at a given time  $t$  and multiply it with the projection matrix  $P$ . After several iterations, typically 20 to 30, we can write  $N_{t+1}$  equal  $P^n$  times  $N_t$ . The projection matrix can be written as  $P^n$ . After several multiplications with itself, the matrix reduces itself to diagonal values of  $\lambda$  called the 'dominant Eigenvalue' of the projection matrix  $P$ . Since  $P$  has now become a diagonal matrix, we can replace it with  $\lambda$  to the power of  $n$ . After several iterations-- 20 to 30, for example-- the population structure becomes stable. The proportions between the vector elements are constant.

The stabilised population vector becomes the Eigenvector. We can standardise the Eigenvector to a length of 1, which is also called the 'normed Eigenvector'. Let us develop a simple matrix model for a population of female cattle, with the following numbers. We have 50,000 calves, 20,000 heifers, and 40,000 cows. The time step is one year. The annual birth rate is 60% calving per year. The survival rate of the calves is 70%, which means 30% die or are taken off per year. Heifers stay two years as heifers. Their persistence rate is 50%. The survival rate of heifers is 60%, and the persistence of cows is 65%. These figures are not real and only for demonstration purposes.

If you don't have a spreadsheet application at hand, you can solve the matrix as follows. To see a projection of the population of the next year  $t$  plus 1, you have to sum up the multiplication of every element of the matrix rows with an element of the vector. Let us do it together.  $0$  times  $Y$  is  $0$ , plus  $0$  times  $S$  is  $0$ , plus  $b$  times  $A$  results in  $bA$ , which is understandable as multiplying the annual birth rate with the number of cows. In the same way, you deal with the other roles. This is a basic matrix calculus, which you may also find in other sources.



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Let's solve the first matrix together.  $Y$  is equal 24,000,  $S$  is equal 45,000, and  $A$  is 38,000. Now, solve the first seven years by using a calculator. Make sure to always multiply the vector of the following year with the same values of the birth rate, the survival rate, and so on. You can download a PDF with the solutions to compare your results. If you have a table calculator programme, you can download the spreadsheet file afterwards and continue the rows to the year 21. What do you notice in this year? And what happens after year 10? Write your insights in the comments section.